

The Red Sea University
College of Graduate Studies & Scientific Research

**Asymptotic Relation and Combinations of Cross-
Products of Bessel Functions and Operator
Systems**

**علاقة المقاربة والدمج للضرب الكارتيزي لدالة بسل
وأنظمة المؤثر**

**A thesis Submitted in Fulfillment of the Requirement for the
Degree of MS.c in Mathematics**

By

Musa Taher Abaker Musa

Supervisor

Dr. Bashra El Fadil Ibrahim

Dedication

To all my family members (my Mother, Father, and all of my brothers and sisters). Specially my beloved wife (Haffsa Hasn Abaker) the geratest person in my life, and my childern (AlaaElden and Maarib).

Acknowledgements

Thanks and gratitude to Allah who completed his blessing for us. Prays and peace up on Mohammed the signet of prophets. The researcher would like to express his gratitude to all who support and advised him, specially the greatest person my supervisor Dr. Bushra Alfadil, his advice and guidances support the researcher in all the research stages. Great thanks to Prof. Dr. Shawgy Hussein Abdalla, Dr. Khalid Abdulslam and Dr. Anusa Abdalrhman, and Red sea University administration, specially faculty of Education "Basic level – department of Mathematics".

Abstract

The crossed products of operator systems and C^* -correspondences by amenable group actions are described. We show some remarks on the zeros and Nicholson-type integral for the cross-product of the Bessel functions. We improve the crossed product correspondences, combinations of cross-products of Bessel functions, associated addition, formulas with C^* -algebras and biorthogonal systems. We study the operator system quotients of matrix algebras, discrete groups and nuclearity by C^* -envelops with their tensor products. The modified extend, finite index Weber transforms, Legendre functions of imaginary degree related to the integral transforms and sharp inequalities for trigonometric sums are characterized. Studies and asymptotic relation for the zeros of cross-product of Bessel functions, orthogonal polynomials and method for their computation are considered.

الخلاصة

قمنا بوصف الضرب الإتجاهي لأنظمة المؤثر وتقابلات- C^* بواسطة أفعال زمر القابلية. تم توضيح بعض الملاحظات على الأصفار والتكامل نوع-نيكولسون لأجل الضرب الاتجاهي لدوال بسل. قمنا بتحسين تقابلات الضرب الاتجاهي والتوافقي للضرب الاتجاهي لدوال بسل وصيغ الاضافة المشاركة مع جبريات - C^* والأنظمة ثنائية المعامدة . درسنا قواسم أنظمة المؤثر لجبريات المصفوفة والزمر المتقطعة والنوعية بواسطة أغلفة - C^* مع ضرب تنسور لها. تم تشخيص التمديد المعدل وتحويلات ويبيير للدليل المنتهي ودوال ليجنيدر للدرجة التخيلية ذات العلاقة إلى تحويلات التكامل والمتباينات القاطعة لمجموعات الهندسة المثلثية. قمنا باعتبار الدراسات وعلاقات المقاربة لأجل الأصفار للضرب الاتجاهي لدوال بسل وكثيرات الحدود المتعامدة والطرق لأجل حوسبتها.

Introduction

For (X, A, φ) be a C^* -correspondence with a continuous action (γ, α) by an amenable locally compact group G . We will show that if $(X_\gamma, A_\alpha, \psi)$ is the induced C^* -correspondence and β is the induced action on O_X , then. If $q : S \rightarrow T$ is a completely positive (cp) linear map of operator systems and if $J = \ker q$, then the quotient vector space S/J may be endowed with a matricial ordering through which S/J has the structure of an operator system. There is a uniquely determined cp map T such that q is the canonical linear map of S onto S/J . The cp map is called a complete quotient map if q is a complete order isomorphism between the operator systems S/J and T . Herein we study certain quotient maps in the cases where S is a full matrix algebra or a full subsystem of tridiagonal matrices. We formulate a general framework for the study of operator systems arising from discrete groups. We study in detail the operator system of the free group F_n on n generators, as well as the operator systems of the free products of finitely many copies of the two-element group Z_2 . We examine various tensor products of group operator systems, including the minimal, the maximal, and the commuting tensor products.

We introduce, specific finite and infinite integral transforms with Bessel functions J° and Y° in their kernels. The infinite transform and its reciprocal look deceptively similar to the known Weber transform and its reciprocal, respectively, but fundamentally differ from them. New integral representations, asymptotic formulas, and series expansions in powers of $\tanh(t/2)$ are obtained for the imaginary and real parts of the Legendre function $P_{i\xi}(\cosh t)$. Coefficients of these series expansions are orthogonal polynomials in the real variable ξ . A number of relations for these orthogonal polynomials are obtained on the basis of the generating function

Several recent results are encountered on the zeros of Bessel functions and of the classical orthogonal polynomials, many of them are waiting for being published.

We show that an operator system is $(\min, \text{ess})$ -nuclear if its C^* -envelope is nuclear. This allows us to deduce that an operator system associated to a generating set of countable discrete group by Farenick et al. is $(\min, \text{ess})$ -nuclear if and only if the group is amenable. When a locally compact group acts on a C^* -correspondence, it also acts on the associated Cuntz-Pimsner algebra in a natural way. Hao and Ng have shown that when the group is amenable the Cuntz-Pimsner algebra of the crossed product correspondence is isomorphic to the crossed product of the Cuntz-Pimsner algebra. We introduce the crossed product construction for a discrete group

action on an operator system. We describe three canonical crossed products arising from such a dynamical system. We describe how these crossed product constructions behave under G -equivariant maps, tensor products, and the canonical C^* -covers.

We deal with a new class of Fredholm integral equations of the first kind associated with Hankel transforms of integer order. Analysis of the equations is based on operators transforming Bessel functions of the first kind into kernels of Weber–Orr integral transforms. Their inverse operators are established by means of new inversion theorems for the Hankel and Weber–Orr integral transforms of functions belonging to L^1 and L^2 . These operators together with the proven Paley–Wiener’s theorem for the Weber–Orr transform enable to regularize the equations and, in special cases, to derive explicit solutions. This study deals with cross-products of Bessel functions of the first and second kinds. Combinations are shown to be generating functions for associated Legendre functions. This fact permits to establish new integral and series representations of. Under special choices of parameters, the representations become integral transforms or coefficients of orthogonal series whose inverting leads to new addition formulas for Bessel functions and their continuous analogues.

The zeros of Bessel functions play an important role in computational mathematics, mathematical physics, and other areas of natural sciences. Studies addressing these zeros (their properties, computational methods) can be found in various sources.

The Contents

Subject	Page
Dedication	I
Acknowledgments	II
Abstract	III
Abstract (Arabic)	IV
Introduction	V
The Contents	VII
Chapter 1 Crossed Products and Operator System Quotients	
Section (1.1): C^* -Correspondences by Amenable Group Actions	1
Section(1.2): Matrix Algebras and Their Tensor Products	6
Section(1.3): Discrete Groups	18
Chapter 2 Modified and Legendre functions with Sharp Inequalities	
Section(2.1): Finite Index Weber Transforms	27
Section(2.2): Imaginary Degree and Associated Integral Transforms	43
Section (2.3): Trigonometric Sums in Two Variables	57
Chapter 3 An Asymptotic Relation and Some Recent Results on the Zeros	
Section(3.1): The Zeros of Cross-Product Bessel Functions	71
Section(3.2): The Zeros of Bessel Functions	76
Section (3.3): The zeros of Bessel Functions and Orthogonal Polynomials	81
Chapter 4 Operator System Nuclearity and Crossed Products	
Section(4.1): C^* -Envelopes	95
Section(4.2): Crossed Product Correspondences and Associated C^* -Algebras	102
Section (4.3): Operator Systems	107
Chapter 5 Certain Class of Integral Equations and A Nicholson-Type Integral and Combinations of Cross-Products	
Section(5.1): Hankel Transforms	120
Section(5.2): The Cross-Product of the Bessel Functions	134
Section(5.3): Bessel Functions with Associated Addition Formulas and Biorthogonal Systems	144
Chapter 6 Studies on the Zeros and Asymptotic Relation	
Section(6.1): Bessel Functions and Methods for Their Computation	162
Section(6.2): Roots of the Bessel and Certain Related Functions	175
Section (6.3): Zeros of Cross-Product of Bessel Functions and Applications	181
List of Symbols	197
References	198

